Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Thursday 14 January 2021

Morning (Time: 1 hour 30 minutes)	Paper Reference **WMA13/01**

Mathematics
International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1/1/1/1/

1. Find

$$\int \frac{x^2 - 5}{2x^3}\,\mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(3)

1. $\int \frac{x^2 - 5}{2x^3}\,dx \qquad x > 0$

$= \int \frac{x^2}{2x^3} - \frac{5}{2x^3}\,dx$

$= \int \frac{1}{2x} - \frac{5}{2x^3}\,dx$

$= \frac{1}{2}\ln(x) + \frac{5}{4}x^{-2} + c$

P 6 6 3 8 9 A 0 2 3 2

2.

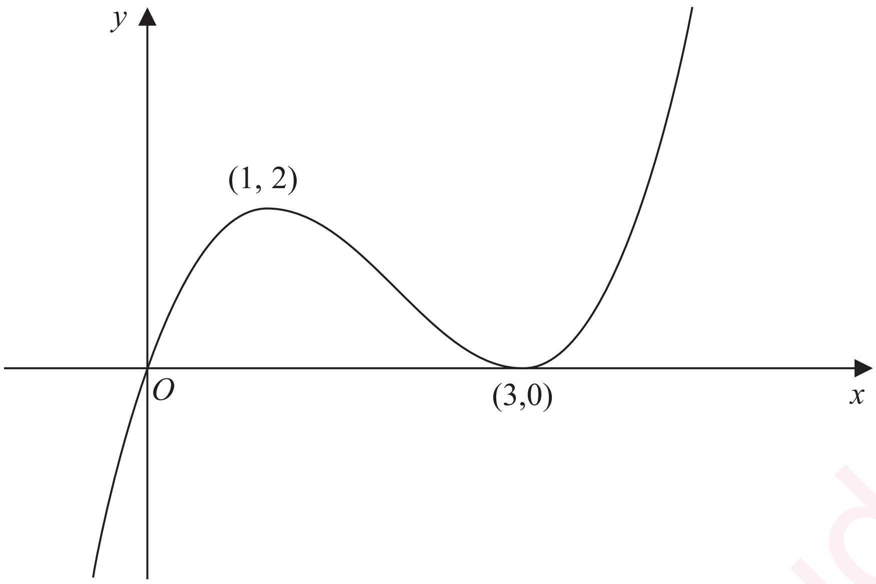


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where $x \in \mathbb{R}$ and $f(x)$ is a polynomial.

The curve passes through the origin and touches the x-axis at the point (3, 0)

There is a maximum turning point at (1, 2) and a minimum turning point at (3, 0)

On separate diagrams, sketch the curve with equation

(i) $y = 3f(2x)$ **(3)**

(ii) $y = f(-x) - 1$ **(3)**

On each sketch, show clearly the coordinates of

- the point where the curve crosses the y-axis
- any maximum or minimum turning points

(i) $y = 3f(2x)$

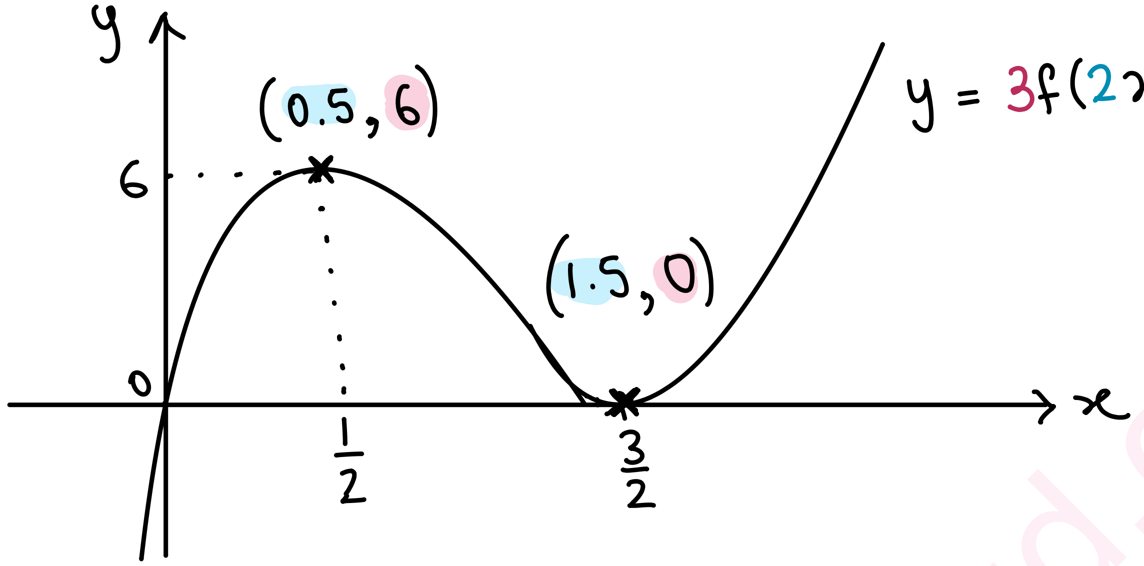
stretch: scale factor 3 parallel to y-axis

stretch: scale factor $\frac{1}{2}$ parallel to x-axis

$\therefore$ turning points : $(1,2) \to \left(1 \times \frac{1}{2}, 2 \times 3\right) = \left(\frac{1}{2}, 6\right)$

$(3,0) \to \left(3 \times \frac{1}{2}, 0 \times 3\right) = \left(\frac{3}{2}, 0\right)$

Question 2 continued



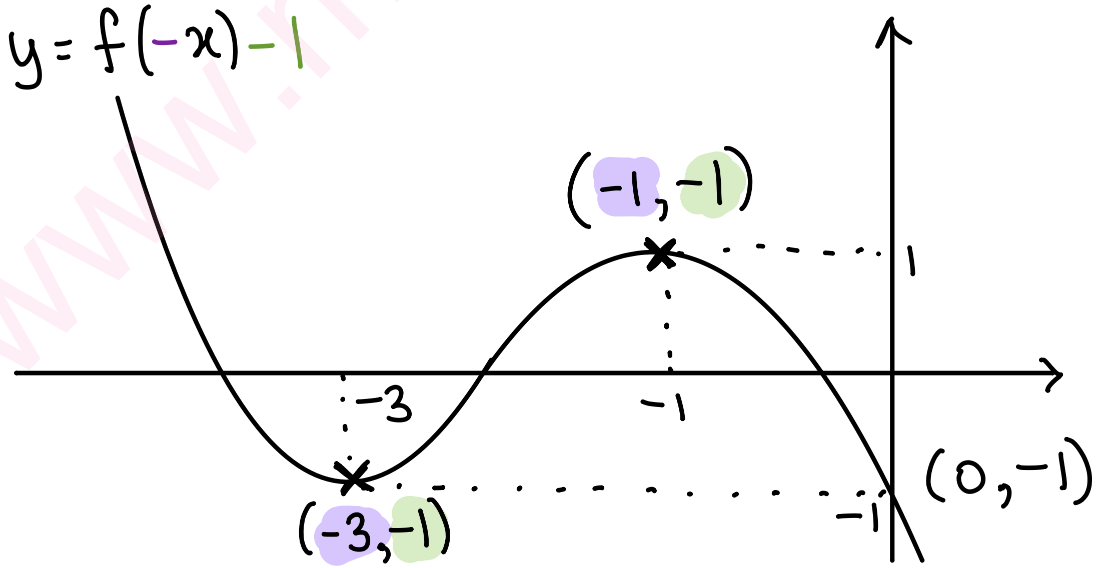
(ii) $y = f(-x) - 1$

reflection in the y-axis

translation through the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\therefore$ turning points : $(1,2) \rightarrow (-(1),\ 2-1) = (-1,-1)$

$(3,0) \rightarrow (-(3),\ 0-1) = (-3,-1)$



(Total 6 marks)

Q2

Turn over

P 6 6 3 8 9 A 0 5 3 2

3.

$$f(x) = 3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} \qquad x > 4$$

(a) Show that

$$f(x) = \frac{ax+b}{cx+d} \qquad x > 4$$

where a, b, c and d are integers to be found.

(4)

(b) Hence find $f^{-1}(x)$

(2)

(c) Find the domain of f^{-1}

(2)

3.a) $f(x) = 3 - \left(\frac{x-2}{x+1}\right) + \frac{5x+26}{2x^2-3x-5}$

$= \frac{3(x+1)-x+2}{x+1} + \frac{5x+26}{(2x-5)(x+1)}$

$= \frac{(2x+5)(2x-5) + 5x+26}{(x+1)(2x-5)}$

$= \frac{4x^2-25+5x+26}{(x+1)(2x-5)}$

$= \frac{4x^2+5x+1}{(x+1)(2x-5)}$

$= \frac{(4x+1)\cancel{(x+1)}}{\cancel{(x+1)}(2x-5)}$

$= \frac{4x+1}{2x-5}$

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Question 3 continued

b) to find $f^{-1}(x)$: $f(x) = \frac{4x+1}{2x-5}$

① write the function using a "y" and set equal to "x" : $x = \frac{4y+1}{2y-5}$

② rearrange to make y the subject : $2xy - 5x = 4y + 1$

$y(2x-4) = 1 + 5x$

③ replace y with $f^{-1}(x)$: $y = \frac{5x+1}{2x-4}$

$\therefore f^{-1}(x) = \frac{5x+1}{2x-4}$

domain of the inverse function :

↑ domain refers to the set of values we are allowed to plug into our function

domain of inverse function = range of function

↑ range refers to all possible values of a function

$\therefore$ domain of $f^{-1}(x)$ = range of $f(x)$

$f(x) = \frac{4x+1}{2x-5} \quad x > 4,$

when $x = 4 \rightarrow f(4) = \frac{4(4)+1}{2(4)-5} = \frac{17}{3}$

$f(x) = \frac{4 + \frac{1}{x}}{2 - \frac{5}{x}} \rightarrow$ as $x \rightarrow \infty \rightarrow f(x) = \frac{4 + \cancel{\frac{1}{x}}^{\,0}}{2 - \cancel{\frac{5}{x}}^{\,0}} \rightarrow \frac{4}{2} = 2$

↑ divide through by x

P 6 6 3 8 9 A 0 7 3 2

Question 3 continued

$\therefore$ range of $f(x)$ is $2 < f(x) < \frac{17}{3}$

$\therefore f^{-1}(x)$ domain is : $2 < x < \frac{17}{3}$

4.

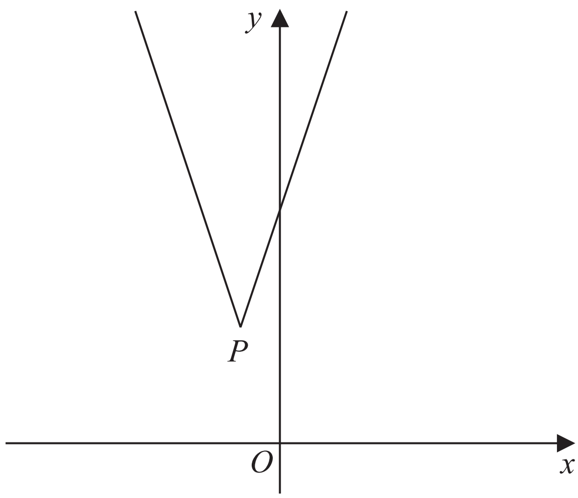


Figure 2

Figure 2 shows a sketch of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |3x + a| + a$$

and where a is a positive constant.

The graph has a vertex at the point P, as shown in Figure 2.

(a) Find, in terms of a, the coordinates of P.

(2)

(b) Sketch the graph with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |x + 5a|$$

On your sketch, show the coordinates, in terms of a, of each point where the graph cuts or meets the coordinate axes.

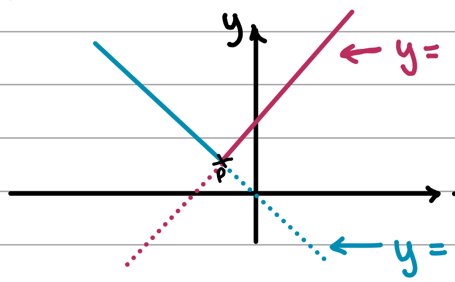
(2)

The graph with equation $y = \mathrm{g}(x)$ intersects the graph with equation $y = \mathrm{f}(x)$ at two points.

(c) Find, in terms of a, the coordinates of the two points.

(5)

4.a) $f(x) = |3x+a| + a$



$\leftarrow y = (3x+a)+a = 3x+2a$

$\leftarrow y = -(3x+a)+a = -3x$

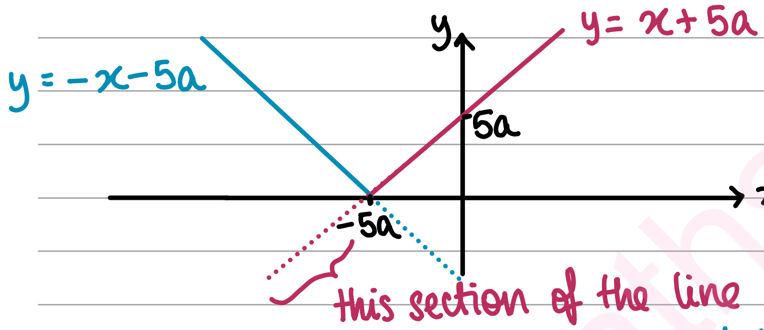
P is the point at which the 2 lines meet

$3x + 2a = -3x$

$6x = -2a$

$x = -\frac{a}{3}$ $\therefore P = \left(-\frac{a}{3}, a\right)$

b) $y = g(x) = |x + 5a|$



this section of the line goes into negative, it is not drawn, the modulus is drawn

c)

$f(x) = |3x + a| + a$
$g(x) = |x + 5a|$

As we can see, both sections of f(x) intersect with the positive section of g(x)

$3x + 2a = x + 5a$ $-3x = x + 5a$

$2x = 3a$ $4x = -5a$

$x = \frac{3a}{2}$ $x = -\frac{5a}{4}$

$\therefore \left(\frac{3a}{2}, \frac{13a}{2}\right)$ & $\left(-\frac{5a}{4}, \frac{15a}{4}\right)$

5. The temperature, θ°C, inside an oven, t minutes after the oven is switched on, is given by

$$\theta = A - 180e^{-kt}$$

where A and k are positive constants.

Given that the temperature inside the oven is initially 18 °C,

(a) find the value of A.

(2)

The temperature inside the oven, 5 minutes after the oven is switched on, is 90 °C.

(b) Show that $k = p\ln q$ where p and q are rational numbers to be found.

(4)

Hence find

(c) the temperature inside the oven 9 minutes after the oven is switched on, giving your answer to 3 significant figures,

(2)

(d) the rate of increase of the temperature inside the oven 9 minutes after the oven is switched on. Give your answer in °C min^{-1} to 3 significant figures.

(3)

5.a) $\theta = A - 180e^{-kt}$

when $t = 0$ → $\theta_0 = A - 180e^{-k(0)}$
$= A - 180 = 18$

$A = 198$

b) when $t = 5$ → $\theta_5 = 198 - 180e^{-k(5)} = 90$

$180e^{-5k} = 108$

$e^{-5k} = \frac{3}{5}$

$k = -\frac{1}{5}\ln\left(\frac{3}{5}\right) = \frac{1}{5}\ln\left(\frac{5}{3}\right)$

$\boxed{\text{Log rule : } a\log_b(c) = \log_b(c^a)}$

c) when $t=9$ → $\theta_9 = 198 - 180e^{-\frac{1}{5}\ln\left(\frac{5}{3}\right)\times 9}$

$= 198 - 71.77...$

$= 126\ °C$

d) rate of increase of temp. $= \frac{d\theta}{dt}$

$\theta = 198 - 180e^{\frac{1}{5}\ln\left(\frac{3}{5}\right)t}$

DIFFERENTIATING EXPONENTIALS

$y = e^{kx} \rightarrow \frac{dy}{dx} = ke^{kx}$

$\frac{d\theta}{dt} = \frac{1}{5}\ln\left(\frac{3}{5}\right) \times -180 \times e^{\frac{1}{5}\ln\left(\frac{3}{5}\right)t}$

$\frac{d\theta}{dt}$ at $t=9$ $= \frac{1}{5}\ln\left(\frac{3}{5}\right) \times -180 \times e^{\frac{1}{5}\ln\left(\frac{3}{5}\right)\times 9}$

$= 7.33\ °C\ \text{min}^{-1}$

6.

$$f(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$$

(a) Find $f'(x)$

(2)

(b) Show that the equation $f'(x) = 0$ can be written as

$$x = k\arctan\left(\frac{k}{x}\right)$$

where k is an integer to be found.

(2)

(c) Starting with $x_1 = 2.5$ use the iteration formula

$$x_{n+1} = k\arctan\left(\frac{k}{x_n}\right)$$

with the value of k found in part (b), to calculate the values of x_2 and x_6 giving your answers to 3 decimal places.

(2)

(d) Using a suitable interval and a suitable function that should be stated, show that a root of $f'(x) = 0$ is 2.581 correct to 3 decimal places.

(2)

6.a) $f(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$

$\boxed{\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'}$

$u = x \qquad \frac{du}{dx} = 1$

$v = \cos\left(\frac{x}{3}\right) \qquad \frac{dv}{dx} = \frac{1}{3} \times -\sin\left(\frac{x}{3}\right)$

$f'(x) = (1)\cos\left(\frac{x}{3}\right) + x\left(\frac{-\sin\left(\frac{x}{3}\right)}{3}\right) = \cos\left(\frac{x}{3}\right) - \frac{x\sin(x/3)}{3}$

if $f'(x) = 0 \qquad \cos\left(\frac{x}{3}\right) - \frac{x\sin(x/3)}{3} = 0$

Question 6 continued

$$x \sin\left(\frac{x}{3}\right) = 3\cos\left(\frac{x}{3}\right)$$

$$\tan\left(\frac{x}{3}\right) = \frac{3}{x}$$

$$\therefore \quad x = 3\arctan\left(\frac{3}{x}\right) \qquad k=3$$

c) $x_{n+1} = 3\arctan\left(\frac{3}{x}\right)$

$x_1 = 2.5$

$x_2 = x_{1+1} = 3\arctan\left(\frac{3}{2.5}\right) = 2.628$

$x_3 = 2.554 \qquad x_5 = 2.572$

$x_4 = 2.597 \qquad x_6 = 2.586$

d) To show 2.581 is correct to 3 d.p.

we must show a sign change between $f'(2.5805)$ $f'(2.5815)$

$f'(2.5805) = \cos\left(\frac{2.5805}{3}\right) - \frac{2.5805}{3}\left(\sin\left(\frac{2.5805}{3}\right)\right) = 0.000346$

$f'(2.5815) = \cos\left(\frac{2.5815}{3}\right) - \frac{2.5815}{3}\left(\sin\left(\frac{2.5815}{3}\right)\right) = -0.000345$

Given that there is a sign change, & the function is continous over the given interval, $\therefore$ 2.581 is a root

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

7. (a) Prove that

$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$7 + \frac{\sin 4\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin 2\theta} = 3\cot^2 2\theta$$

giving your answers in radians to 3 significant figures where appropriate.

(6)

7. a) LHS : $\frac{\sin(2x)}{\cos(x)} + \frac{\cos(2x)}{\sin(x)}$ RHS : $\operatorname{cosec}(x) = \frac{1}{\sin(x)}$

USING COMPOUND ANGLE FORMULAE → $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$= \frac{2\sin(x)\cancel{\cos(x)}}{\cancel{\cos(x)}} + \frac{\cos^2(x)-\sin^2(x)}{\sin(x)}$

$= 2\sin(x) + \frac{\cos^2(x)-\sin^2(x)}{\sin(x)}$

$= \frac{2\sin^2(x)+\cos^2(x)-\sin^2(x)}{\sin(x)}$

$= \frac{\sin^2(x)+\cos^2(x)}{\sin(x)}$ ← $\sin^2(A)+\cos^2(A) = 1$

$= \frac{1}{\sin(x)} = \operatorname{cosec}(x) = \text{RHS}$

Question 7 continued

b) $7 + \frac{\sin(4\theta)}{\cos(2\theta)} + \frac{\cos(4\theta)}{\sin(2\theta)} = 3\cot^2(2\theta)$

$7 + \frac{1}{\sin(2\theta)} = \frac{3\cos^2(2\theta)}{\sin^2(2\theta)}$

$7\sin^2(2\theta) + \sin(2\theta) = 3\cos^2(2\theta)$ $[\cos^2(2A) = 1 - \sin^2(2A)]$

$7\sin^2(2\theta) + \sin(2\theta) = 3 - 3\sin^2(2\theta)$

$10\sin^2(2\theta) + \sin(2\theta) - 3 = 0$

let $\sin(2\theta) = w$

$10w^2 + w - 3 = 0$

$(5w+3)(2w-1) = 0$

$w = -\frac{3}{5} \cup \frac{1}{2}$

$\therefore \sin(2\theta) = -\frac{3}{5} \cup \frac{1}{2}$

$2\theta = -0.644 \cup \pi/6$
$\cup -2.50 \cup 5\pi/6$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$-\pi < 2\theta < \pi$

$\theta = -0.322 \cup \pi/12$
$\cup -1.25 \cup 5\pi/12$

8. The percentage, P, of the population of a small country who have access to the internet, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after the start of 2005

Using the data for the years between the start of 2005 and the start of 2010, a graph is plotted of $\log_{10} P$ against t.

The points are found to lie approximately on a straight line with gradient 0.09 and intercept 0.68 on the $\log_{10} P$ axis.

(a) Find, according to the model, the value of a and the value of b, giving your answers to 2 decimal places.

(4)

(b) In the context of the model, give a practical interpretation of the constant a.

(1)

(c) Use the model to estimate the percentage of the population who had access to the internet at the start of 2015

(2)

LOG RULES

$a\log_b(c) = \log_b(c^a)$

$\log_a b + \log_a c = \log_a(bc)$

$\log_a b = c \rightarrow a^c = b$

8. a) $P = ab^t$

$\log_{10} P = \log_{10}(ab^t)$

$\log_{10} P = \log_{10} a + \log_{10}(b^t)$

$\log_{10} P = \log_{10} a + t\log_{10} b$ } graph of $\log_{10} P$ against t

$y = c + x\,m$

gradient $= 0.09 = m = \log_{10} b$

$b = 10^{0.09} = 1.23$

intercept $= 0.68 = c = \log_{10} a$

$a = 10^{0.68} = 4.79$

b) $P = ab^t = 4.79 \times 1.23^t$

when $t = 0 \rightarrow P = 4.79 \times 1.23^0 = 4.79$

$\therefore$ a is percentage of population with access at the start of 2005

Question 8 continued

c) $P = ab^{10} = 4.79 \times 1.23^{10} = 38\ \%$

(Total 7 marks)

Q8

9. Find

(i) $\displaystyle\int \frac{3x-2}{3x^2-4x+5}\,dx$

(2)

(ii) $\displaystyle\int \frac{e^{2x}}{(e^{2x}-1)^3}\,dx \qquad x \neq 0$

(2)

9. (i) $\displaystyle\int \frac{3x-2}{3x^2-4x+5}\,dx$

let $3x^2-4x+5 = u$

$\frac{du}{dx} = 6x-4 \rightarrow dx = \frac{du}{6x-4}$

$\therefore \displaystyle\int \frac{3x-2}{u}\,dx = \int \frac{3x-2}{u} \times \frac{du}{6x-4}$

$= \displaystyle\int \frac{\cancel{(3x-2)}}{2\cancel{(3x-2)}\,u}\,du$

$= \frac{1}{2}\displaystyle\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln(u) + c$

$= \frac{1}{2}\ln(3x^2-4x+5) + c$

(ii) $\displaystyle\int \frac{e^{2x}}{(e^{2x}-1)^3}\,dx \qquad x \neq 0$

let $u = e^{2x}-1$

$\frac{du}{dx} = 2e^{2x} \rightarrow dx = \frac{du}{2e^{2x}}$

Question 9 continued

$$\therefore \int \frac{\cancel{e^{2x}}}{u^3} \times \frac{du}{2\cancel{e^{2x}}}$$

$$= \frac{1}{2}\int u^{-3}\ du$$

$$= \frac{1}{2}\left(-\frac{1}{2}u^{-2}\right) + c$$

$$= -\frac{1}{4u^2} + c$$

$$= -\frac{1}{4(e^{2x}-1)^2} + c$$

(Total 4 marks)

Q9

10. The curve C has equation

$$x = 3\sec^2 2y \qquad x > 3 \qquad 0 < y < \frac{\pi}{4}$$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

(2)

(b) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{p}{qx\sqrt{x-3}}$$

where p is irrational and q is an integer, stating the values of p and q.

(3)

(c) Find the equation of the normal to C at the point where $y = \frac{\pi}{12}$, giving your answer in the form $y = mx + c$, giving m and c as exact irrational numbers.

(5)

10. a) $x = 3\sec^2(2y)$

$\boxed{\text{CHAIN RULE : } \frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}}$

$v = \sec(2y)$ $\qquad \frac{dv}{dy} = 2 \times \sec(2y) \times \tan(2y)$

$\therefore x = 3v^2$ $\qquad \frac{dx}{dv} = 6v$

$\frac{dx}{dy} = \frac{dx}{dv} \times \frac{dv}{dy}$

$= 6v \times 2\sec(2y)\tan(2y)$

$= 12\sec^2(2y)\tan(2y)$

b) $\boxed{\frac{dy}{dx} = \frac{1}{(dx/dy)}}$ $\qquad \frac{dy}{dx} = \frac{1}{dx/dy}$

P 6 6 3 8 9 A 0 3 0 3 2

Question 10 continued

$x = 3\sec^2(2y)$

$\sin^2 A + \cos^2 A = 1 \leftarrow$ divide through by $\cos^2 A$

$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$

$\boxed{\tan^2 A + 1 = \sec^2 A}$

$\frac{dx}{dy} = 12\sec^2(2y)\tan(2y)$

$= 12\sec^2(2y)\sqrt{\sec^2(2y) - 1} \leftarrow$ now substitute x

$= 12\left(\frac{x}{3}\right)\sqrt{\frac{x}{3} - 1}$

$= 4x\sqrt{\frac{x}{3} - 1}$

$\therefore \frac{dy}{dx} = \frac{1}{4x\sqrt{x/3 - 1}} = \frac{1}{\frac{4}{\sqrt{3}}x\sqrt{x-3}} = \frac{\sqrt{3}}{4x\sqrt{x-3}}$

c) $y = \frac{\pi}{12}$ $\quad x = 3\sec^2\left(\frac{2\pi}{12}\right) = 4$

gradient of tangent at C $= \frac{dy}{dx} = \frac{\sqrt{3}}{4(4)\sqrt{(4)-3}} = \frac{\sqrt{3}}{16}$

$\therefore$ gradient$_{\text{normal}} = \frac{-1}{\text{grad. tangent}} = \frac{-16}{\sqrt{3}}$

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Question 10 continued

Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line

gradient

→ Known point is C

→ $\text{grad.}_{nor} = -\frac{16}{\sqrt{3}}$

$$y - \frac{\pi}{12} = -\frac{16}{\sqrt{3}}(x - 4)$$

$$y = -\frac{16\sqrt{3}}{3}x + \frac{64\sqrt{3}}{3} + \frac{\pi}{12}$$

Q10

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END